The Double Integral of a function over a Rectangle R.

Definition: Given rectangle R,

$$R = [a,b] \times [c,d],$$

The Double Integral of f over Rectangle R is the limit

$$\iint_R f(x,y)\,dA = \lim_{\substack{m\to\infty \\ n\to\infty}} \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\,\Delta A_{ij}$$

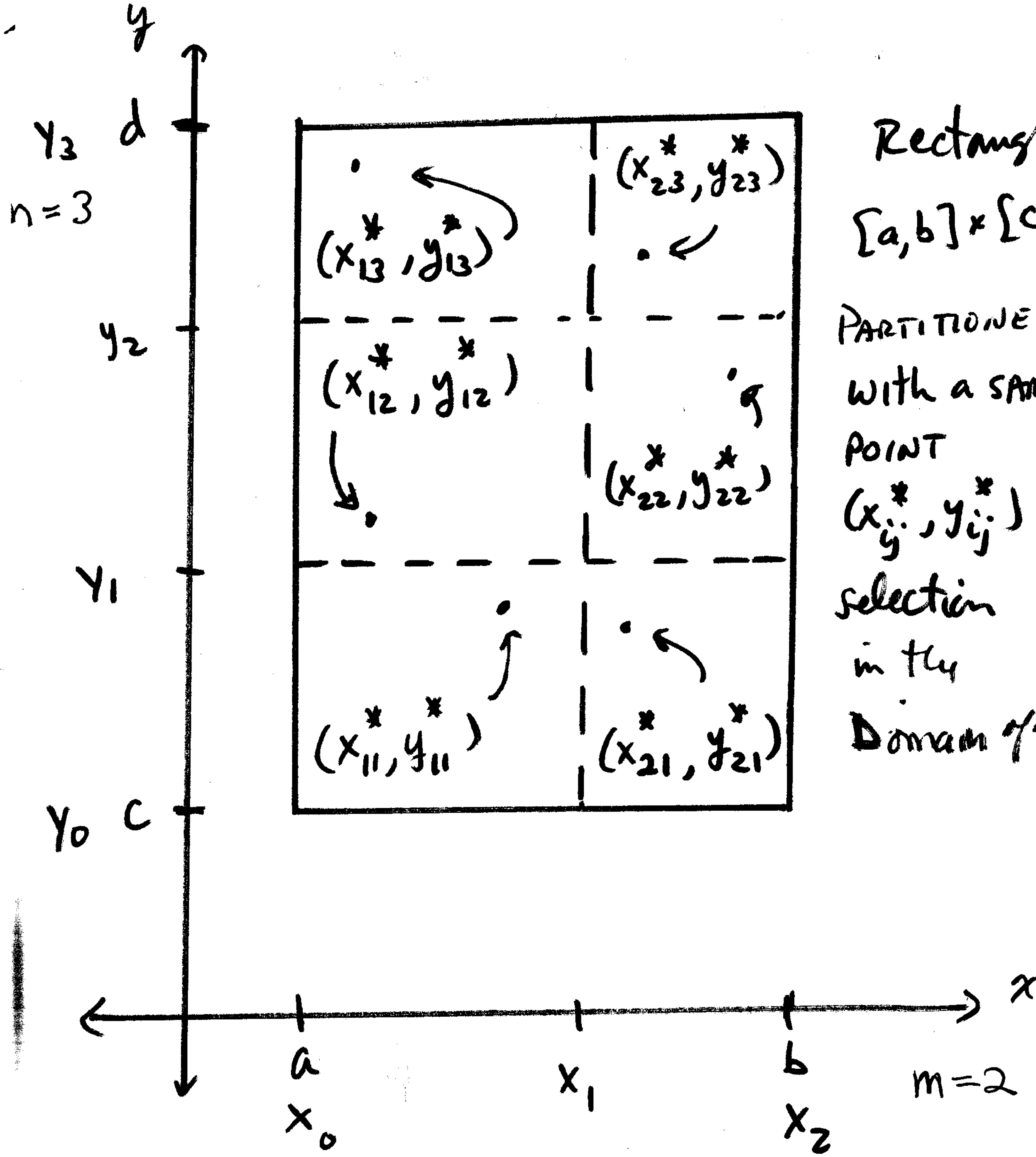
when this limit exists.

When $f(x,y) \geq 0$ for every point (x,y) in Rectangle R, then

① The graph $z = f(x,y)$ above the Rectangle R lies totally above the xy plane. and

② The Double Integral equals the Volume V above the Rectangle and below the graph surface.

$$\text{Volume } V = \iint_R f(x,y)\,dA$$



Rectangle $[a,b] \times [c,d]$ PARTITIONED with a SAMPLE POINT (x^*_{ij}, y^*_{ij}) selection in the Domain of f

Form the Riemann Sum

$$\sum_{i=1}^{2} \sum_{j=1}^{3} f(x^*_{ij}, y^*_{ij}) \cdot \Delta A_{ij}$$

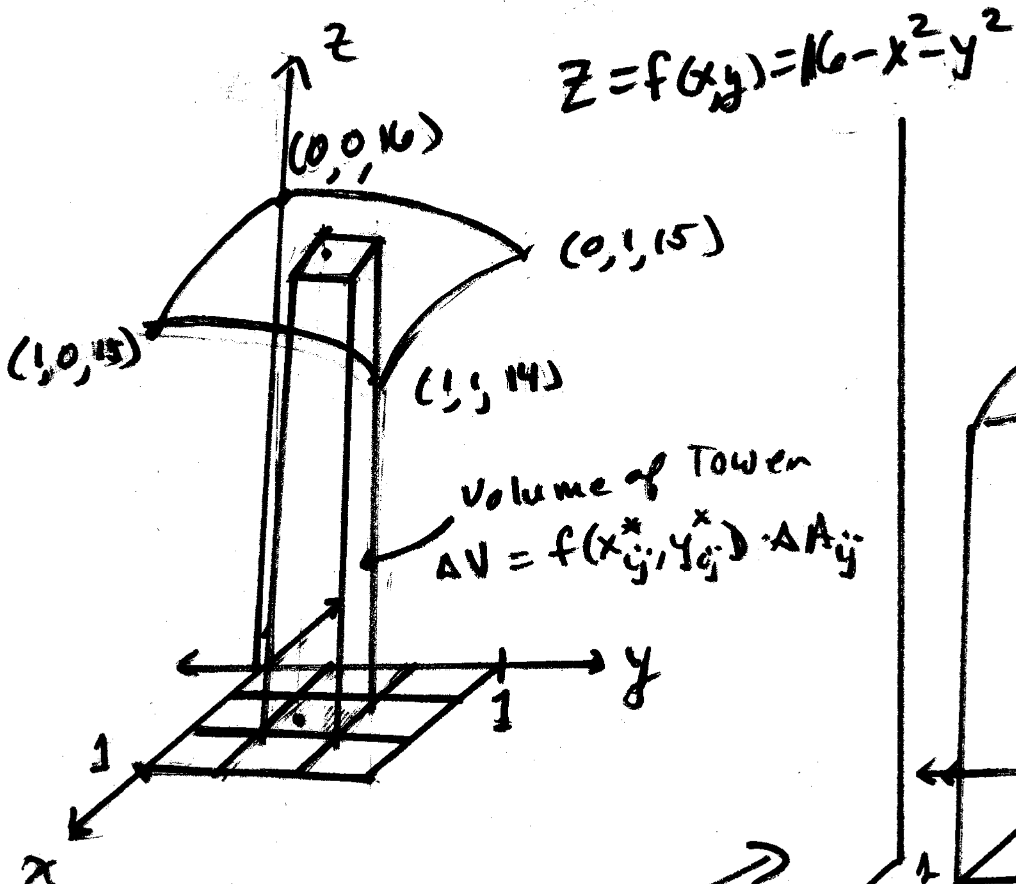
Area of Sub-Rectangle ij.

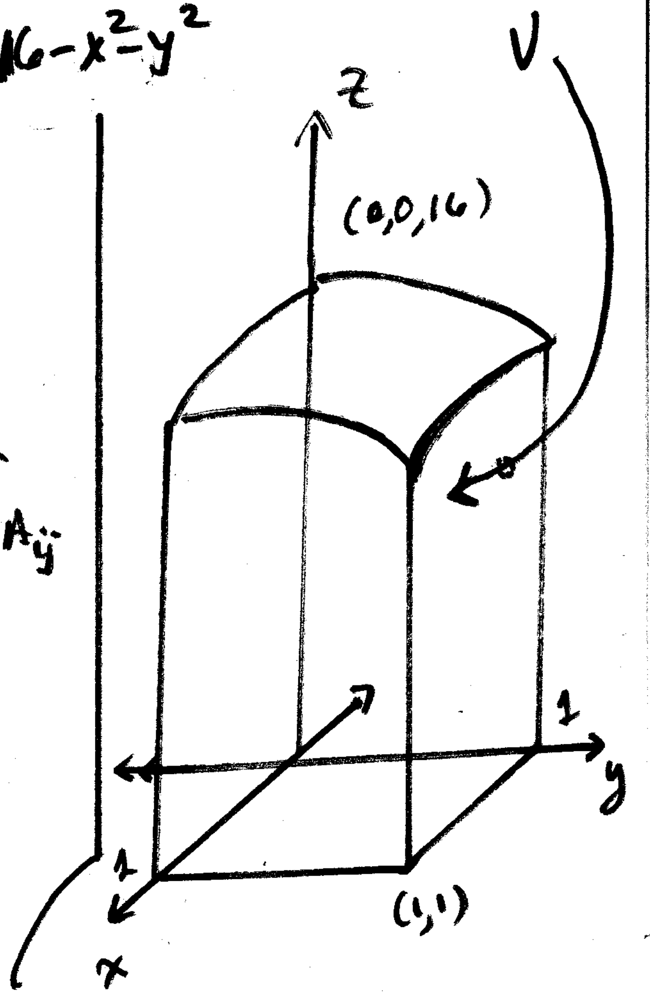
Consider $f(x,y) = 16 - x^2 - y^2$.

Consider Rectangle $R = [0,1] \times [0,1]$

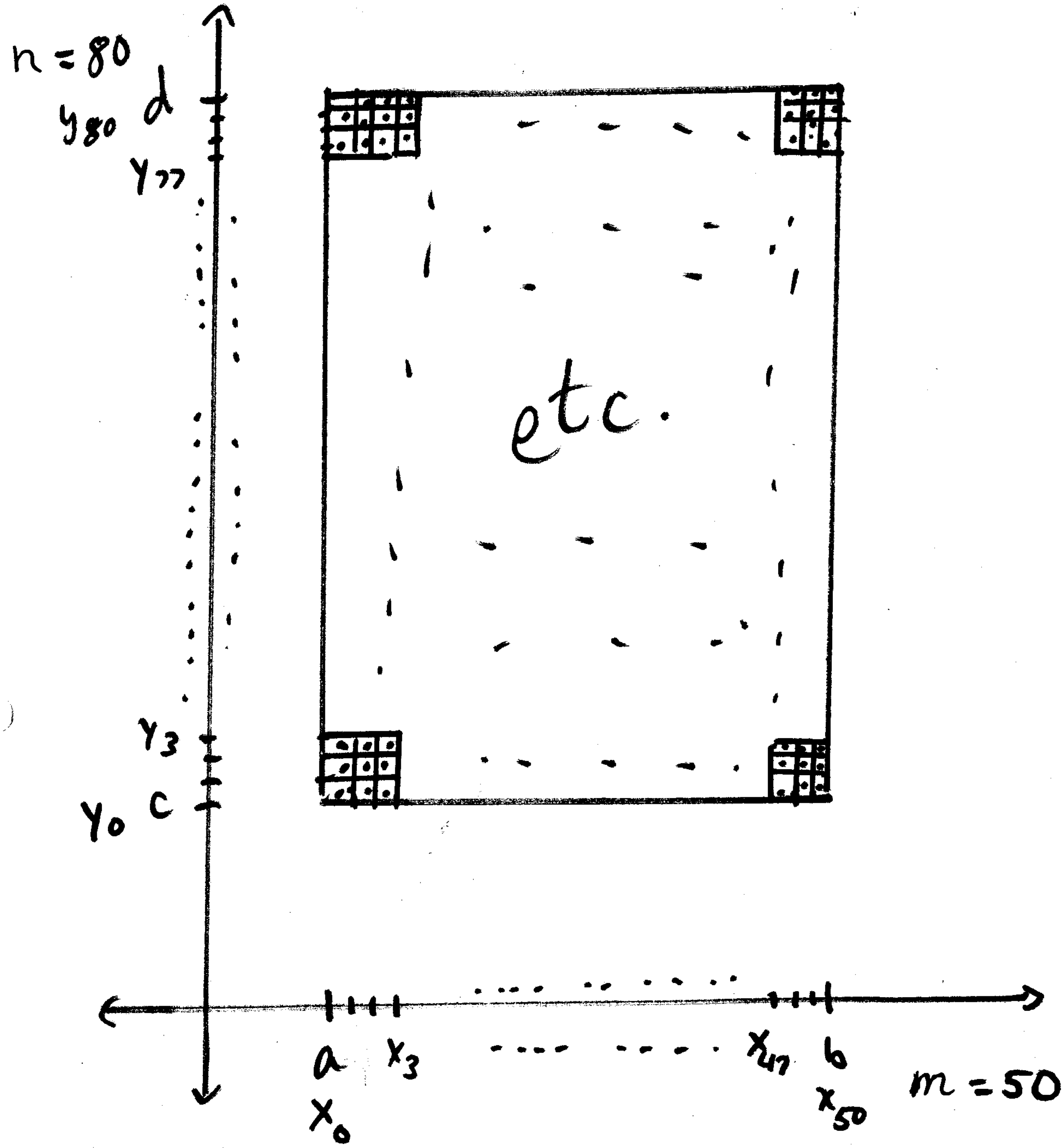
Consider a single tower built upon one subrectangle in a Riemann Sum.

Compare this with the final Limit of these sums, Limit $= \iint_R f(x,y)\,dA = \iint_R (16-x^2-y^2)\,dA$





$V = \iint_R f(x,y)\,dA$



Form the RIEMANN SUM:

$$\sum_{i=1}^{50} \sum_{j=1}^{80} f(x_{ij}^*, y_{ij}^*) \cdot \Delta A_{ij}$$

Area of Sub-Rectangle ij.